Softstuf, Inc.

Calculation of the RMS of a Continuous Sinusoidal Wave Over Various Intervals

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Summary

Interval	*RMS Value*
$0 \rightarrow T$	$I_{Max.}(0.707)$
$0 \rightarrow 0.5T$	$I_{Max.}(0.707)$
$0 \rightarrow 0.25T$	$I_{Max.}(0.707)$
$0 \rightarrow 0.125T$	$I_{Max.}(0.426)$
$0.1T \rightarrow 0.35T$	$I_{Max.}(0.896)$
$0.3T \rightarrow 0.55T$	$I_{Max.}(0.559)$
$0.7T \rightarrow 0.95T$	$I_{Max.}(0.829)$
$0.25T \rightarrow 0.375T$	$I_{Max.}(0.905)$
$0.8T \rightarrow 0.925T$	$I_{Max.}(0.755)$

RMS Formula for a Continuous Sinusoidal Function

$$RMS = \sqrt{\frac{1}{b-a}\int_{a}^{b}[f(x)]^2\ dx}$$

$$= \sqrt{\frac{1}{x_2T - x_1T}\int_{x_1T}^{x_2T}[\ I_{Max.}\sin(\omega t)\ ]^2\ dt} \quad \text{where } \omega = \text{Angular Frequency and } T = \text{Period}$$

$$\text{Power Reducing Formula:} \quad sin^2u = \frac{1-cos2u}{2}$$

$$= \sqrt{\frac{{I_{Max.}}^2}{T(x_2 - x_1)}\int_{x_1T}^{x_2T}\frac{1-\cos(2\omega t)}{2}\ dt}$$

$$= \sqrt{\frac{{I_{Max.}}^2}{2T(x_2 - x_1)}\left[t - \frac{1}{2\omega}\sin(2\omega t)\right]_{x_1T}^{x_2T}}$$

$$= \sqrt{\frac{{I_{Max.}}^2}{2T(x_2 - x_1)}\left[\left(x_2T - \frac{1}{2\omega}\sin(2\omega x_2T)\right) - \left(\ x_1T - \frac{1}{2\omega}\sin(2\omega x_1T)\right)\right]}$$

$$\text{Angular Frequency:} \quad \omega = \frac{2\pi}{T}$$

$$= \sqrt{\frac{{I_{Max.}}^2}{2T(x_2 - x_1)}\left[\left(x_2T - \frac{T}{4\pi}\sin\left(\frac{4\pi x_2T}{T}\right)\right) - \left(x_1T - \frac{T}{4\pi}\sin\left(\frac{4\pi x_1T}{T}\right)\right)\right]}$$

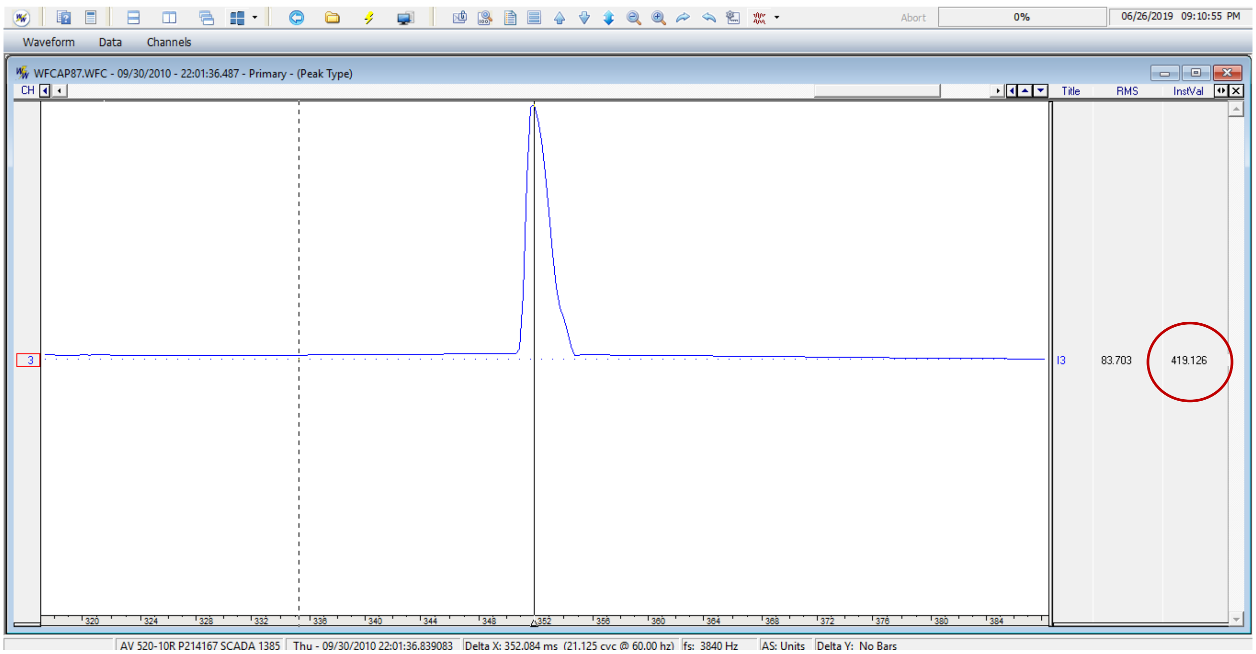
$$= I_{Max.}\sqrt{\frac{4\pi x_2 - \sin(4\pi x_2) - 4\pi x_1 + \sin(4\pi x_1)}{8\pi(x_2 - x_1)}}$$

The above equation is useful when approximating the RMS value of a sinusoidal waveform over various intervals. For the interval $x_1 = 0$ to $x_2 = 1$ the equation above simplifies to the well-known equation $I_{Max.}/\sqrt{2}$. However, for an interval that does not contain the peak of the waveform such as $x_1 = 0$ to $x_2 = 1/8$, the equation simplifies to $I_{Max.}\sqrt{(\pi - 2)/2\pi}$. Furthermore, intervals that contain the peak but do not start from zero also do not simplify to the well-known RMS equation. Because of this it is important to calculate the RMS value using the true discrete formula that is shown on the next page.

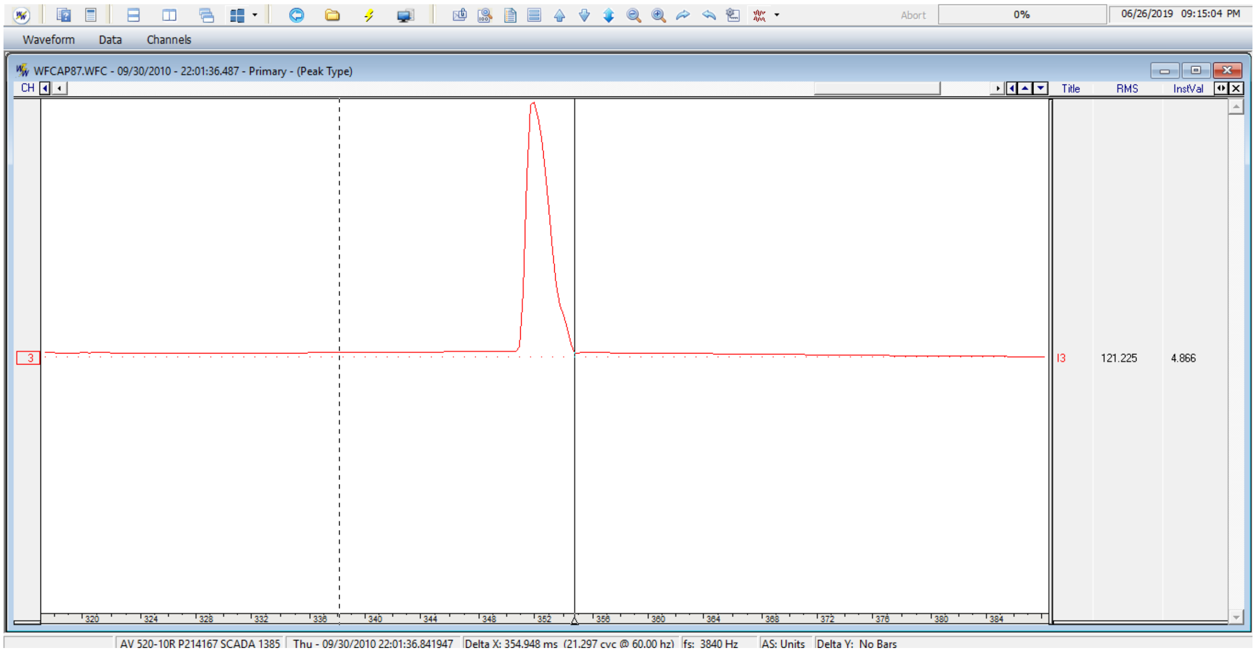
$$I_{RMS} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} {x_1}^2}$$

This formula calculates the RMS value using each data sample rather than approximating the waveform as a sinusoid. The equation above therefore produces a more precise RMS value than the equation shown on the previous page. The true RMS value can be calculated using Wavewin as shown below.

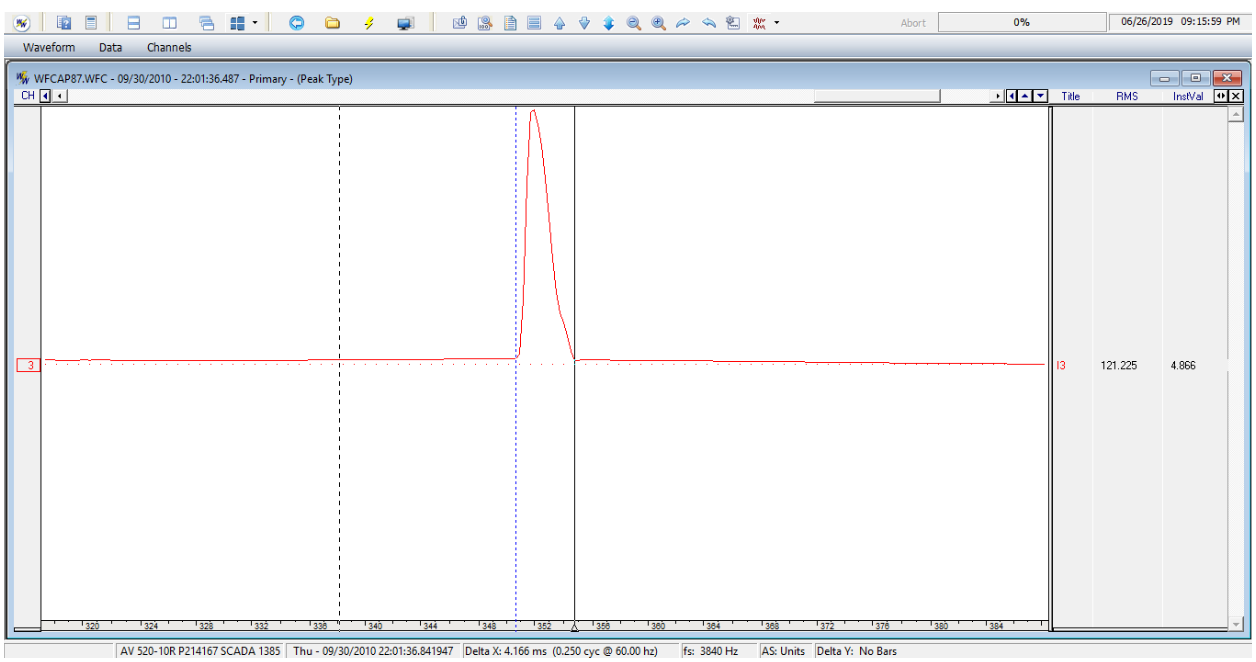
Step 1: Isolate the waveform.



Step 2: Normal click at the end of the pulse.



Step 3: Opposite click at the beginning of the pulse.



Step 4: In the Waveform menu, click Set RMS Bar and read the RMS value.